Distinguishing fluctuation from flow in the fluctuation-dissipation theorem

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The fluctuation-dissipation theorem is unique in physics by relating an equilibrium property to dissipation. Derivations of this theorem confuse the roles of fluctuation and mean flow in thermodynamics and introduce dissipation only by assumption. This is a mild issue for understanding Brownian motion. Yet it strikes at the heart of Lars Onsager's argument concluding that reciprocal macroscopic flows must be equal in the quasi-equilibrium limit because correlation of fluctuations is time reversible. In doing so, he relates irreversible thermodynamic flows to equilibrium fluctuations, apparently providing the final crucial bridge to secure statistical mechanics as a dynamic theory. Onsager's derivation implies, however, that entropy should increase continually, even in equilibrium, by following a fluctuation-dissipation equation. This contradiction is resolved by quantum mode analysis that rigorously distinguishes mean flow and random fluctuation and provides a consistent basis for understanding irreversibility and thermodynamics generally.

Keywords: fluctuation, dissipation, irreversibility, reciprocal relations, Boltzmann principle, Onsager extremal principle, Nyquist relation, Brownian motion

I. INTRODUCTION

John Johnson observed that the electronic noise across a resistor measured by a sensitive amplifier is proportional to both its resistance and temperature, independent of the resistor composition [1]. Soon after, Harry Nyquist derived a formula matching these characteristics by invoking the Second Law of thermodynamics [2]. Lars Onsager followed with a general derivation relating thermodynamic fluctuation and dissipation, implying that flows due to different properties of a system prepared under reciprocal conditions must be equal [3, 4].

"The Nyquist relation is thus of a form unique in physics, correlating a property of a system in equilibrium (i.e. the voltage fluctuations) with a parameter which characterizes an irreversible process (i.e. electrical resistance)" [5, p. 34]. The fluctuation-dissipation theorem (FDT) "is a generalization of statistical mechanics which affords exact formulation as the basis of calculation of such irreversible quantities from atomistic theory" [6, p. 570]. "The fluctuation-dissipation theorem can thus be used in two ways: it can predict the characteristics of the fluctuation or the noise intrinsic to the system from the known characteristics of the admittance or the impedance, or it can be used as the basic formula to derive the admittance from the analysis of thermal fluctuations of the system. The Nyquist theorem is a classical example of the first category (Nyquist 1928), whereas, perhaps, Onsager's proof of the symmetry of kinetic coefficients is the oldest example of the second (Onsager 1931)" [7, p. 256].

These statements are unique in their association of equilibrium properties with dissipation but in no instance is the association derived solely from fundamental physical principles. Each relies on the Second Law postulate and empirically observed patterns, expressed as transport laws of nature, to relate fluctuation amplitude to macroscopic dissipative processes. "We now know that the Second Law of Thermodynamics can be derived assuming ergodicity at equilibrium, and causality. We take the assumption of causality to be axiomatic. It is causality which ultimately is responsible for breaking time reversal symmetry and which leads to the possibility of irreversible macroscopic behaviour" [8, p. 1529]. But as used here, causality is the omission of reverse time events, which evades explaining the time-symmetry of forces that originally inspired the need for entropy. The problem is swept by this reasoning from under one proverbial rug to another.

Onsager's conclusion ostensibly represents the missing driver of irreversible processes in otherwise static classical and statistical mechanical theories of equilibrium. Rigorous distinction between fluctuation and bulk flows reveals the limitations of these statements.

II. RECIPROCAL RELATIONS

In his 1931 paper, Lars Onsager aims to explain generally why certain flows are observed to be closely related near equilibrium [3]. He derives analogs of the FDT and Lord Rayleigh's principle of least macroscopic dissipation of energy with thermodynamic forces replacing mechanical ones. His result suggests this reciprocal flow behavior is consistent with the Second Law, justifying the use of empirical transport laws in near-equilibrium analysis of irreversible processes. He reasons as follows.

Onsager considers "aged" systems that have settled to an equilibrium state characterized by detailed balance in which the only action to be analyzed is the continual fluctuation of local parameter values occurring about the steady average configuration. He states that "We have assumed microscopic [time] reversibility, and at the same time we have assumed that the average decay of fluctuations will obey the ordinary laws of heat conduction" [3, p. 418]. He claims that "there is no logical contradic-

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tion ... [by] neglecting the time needed for acceleration of the heat flow. This time is probably rather small, e.g. in gases it ought to be of the same order of magnitude as the average time spent by a molecule between two collisions. For practical purposes the time-lag can be neglected in all cases of heat conduction that are likely to be studied, and this approximation is always involved in the formulation of laws like [heat conduction]" [3, p. 419]. Therefore, only forward time after excitation is relevant because fluctuation is zero prior. He also implicitly assumes the Second Law and ergodicity: "In order to calculate [thermodynamic probability] one needs a complete (molecular) theory of the system in hand. ... (we must of course assume that these laws do permit statistical equilibrium of some kind). ... We expect that such a system will in the course of time pass through all the (thermodynamic) states" [4, p. 2270].

Onsager defines the thermodynamic force associated with a parameter as proportional to the spatial gradient of the mean parameter displacement from the mean value. The rate of change in mean displacement then indicates flow. By assuming that empirical transport laws of macroscopic flows also apply to fluctuations, this flow may be expressed as a linear combination of thermodynamic forces. Microscopic reversibility of equilibrium fluctuations in a homogeneous material implies that the flow associated with one parameter triggered by an earlier fluctuation of another equals the flow associated with that other parameter when their order is reversed. If fluctuations can be modeled generally by the Boltzmann and Gibbs principles, relating entropy and state probability, even when their mechanism is not known, and if entropy is an analytic function of the various displacement modes, then the force coefficient matrix is symmetric, revealing reciprocal relations between flows from all off-diagonal pairs of forces.

Onsager follows Boltzmann in associating probability of thermodynamic states with well-defined residence time and entropy proportional to the average range of phase space a state encompasses. He also adopts Josiah Gibbs' conjecture that entropy is a dynamic property well-defined out equilibrium [9] and assumes that a system fluctuates among such lower probability states. He thereby joins equilibrium and nonequilibrium dynamics.

Onsager then associates the local accumulation of heat implied by flows with the local change in entropy. Total system entropy grows at a maximal rate equal to the heat inflow through the local boundary plus internal dissipation, defined as the product of thermodynamic force and its conjugate flux, which must be positive to conform to the Second Law. This result, known as Onsager's thermodynamic extremal principle, appears to validate the Second Law as representing irreversible processes.

On further inspection, though, there is a basic issue undermining this interpretation: If fluctuations dissipate energy like macroscopic flows, then entropy of a system in equilibrium increases unceasingly because fluctuations are ever present and uncorrelated. Dissipation is positive in (5.10) of [3] and (5.8) of [4] and Chapter 8 (161) of [10] with no heat flow or applied driving force. Continual growth would contradict the definition of entropy as a state function and therefore steady in equilibrium.

At first glance, this issue is consistent with Onsager's assumption that entropy is maximum for a specific state so that all fluctuations are then out of equilibrium with lower entropy. It is then logical that such nonequilibrium states would evolve on average to equilibrium according to natural laws observed under similar controlled conditions. Imagine entropy as a hilltop with state as a dot situated at the peak in equilibrium. Each quick acceleration away from the peak draws energy into the system which then dissipates gradually within the system as the dot regresses back to the peak. Energy and entropy ratchet up continually. But then the crucial question is how are fluctuations suddenly accelerated from equilibrium and what is the energy source? Neglect of fluctuation acceleration does not avoid the contradiction of mixing reversible physics and irreversible empirical models without an underlying theory consistently producing both effects.

The problem is that irreversible dissipation due to local mean flows should not be confused with microscopic fluctuations about the mean properties. Flows are associated with persistent transfer of particles, energy and momentum due to differences between mean properties in adjacent subsystems that satisfy the thermodynamic limit. This mean transfer is permanent without some subsequent outside disruption.

There is no separate power source driving fluctuations. Random collisions and quantum jumps shuffle microscopic energy locally, disrupting any coincidental motion just as quickly as it forms. This shuffling is a signature of diffusion yet has no effect on mean flows or equilibrium properties [11]. In effect, it converts heat into heat, which does not change the equilibrium state because "All heat is of the same kind," according to J.C. Maxwell [12]. In other words, fluctuations do not generate net mean energy transfer. There is not sufficient power in fluctuations to drive irreversible behavior. They do not drive a system out of stable equilibrium!

Classical thermodynamics represents such processes as heat exchange between two subsystems with zero change in entropy of an isolated aged system. In Gibbs' view, the entropy of each subsystem fluctuates about a steady mean value as heat shuffles back and forth. The entropy of one subsystem necessarily drops when the other increases, which does not contradict the Second Law if it is understood that the Second Law only pertains to mean behavior and that no irreversible process has occurred. The macrostate of this system wanders due to fluctuations with equal residence time through a stationary region of phase space that is a maximum consistent with constraints. The hilltop in this case is a flat mesa whose rim defines equilibrium. Any process over the rim requires a persistent mean shift in conditions to push the system out of equilibrium.

Entropy growth due to dissipation can only be understood as heat input from a macroscopic energy reservoir “external” to (i.e. excluded from) the system “internal energy,” which includes only heat content and formation energy [13]. Dissipation represents the conversion of a non-thermal energy source into heat. For example, macroscopic kinetic energy is the source for friction. Any initial mean motion is damped to zero without a power source to drive it. All such sources are depleted in equilibrium. Entropy growth from dissipation therefore should stop when there is no net mean flow among any subsystems.

For system entropy to be stable in equilibrium under Onsager’s assumptions, fluctuation growth would have to reduce entropy to compensate for entropy growth during regression, which is clearly not a negligible issue and undermines argument that fluctuations represent irreversible processes. Fluctuation growth implies greater correlation and therefore greater local macroscopic energy, which then dissipates back into heat. Such growth can only come from the local heat content, representing a local internal entropy sink $ds = dQ/T < 0$ as heat is lost during this period. But the Second Law requires internal entropy production everywhere, expressed as positive entropy source rate density $ds \geq 0$ [10].

There are several relevant points to make here regarding rapid acceleration. First, dynamic asymmetry imposes a preferred direction of time; fluctuations evidently would grow smoothly and decay suddenly under time reversal. Second, fluctuation acceleration that occurs within one or even a few collision times requires a strong, impulsive force by highly aligned particles to drive flow of many particles. On one hand, regular occurrence of such improbable configurations, and the implied drop in entropy, should be as significant as gradual regression to the mean. On the other hand, weak impulses driving a few particles can hardly be characterized by macroscopic laws. Third, the FDT implies that dissipation is proportional to the power spectrum of the fluctuation amplitude. Rapid rise in amplitude should not be neglected because it represents a significant high frequency portion of this spectrum. Fourth, if fluctuations obey the same laws as the corresponding macroscopic irreversible processes, then thermodynamic forces resist any change; growth and regression rates should be comparable.

How elemental physics processes produce irreversible processes is the long sought missing link in this context. Yet Onsager justifies assuming both microscopic reversibility and fluctuation decay with “The premises and the consequences of Boltzmann’s principle [Entropy $S = k_{\mathrm{B}} \log W +$ Constant for state probability W] have been discussed by A. Einstein to an extent which will be practically sufficient for our purposes” [4, p. 2270].

In the paper Onsager cites [14], Albert Einstein reasons that system state probability is inherent and entropy is defined through Boltzmann’s principle. Probability is defined according to Boltzmann as residence time in a phase space element over an extremely long observation time such that all accessible states are traversed sufficiently to establish a stable time for each element. Einstein attributes the “apparent irreversibility” of physical processes to the tendency of a system to evolve to higher probability states and that the equilibrium state has far higher probability than all others. Onsager infers that fluctuations are states which follow Boltzmann’s principle and therefore evolve in an apparently irreversible manner.

How systems evolve from trajectories of lower to higher probability, i.e. how can processes simply appear irreversible in a unitary theory, is another way of stating the very problem that thermodynamics theory must resolve. Simply claiming that it happens provides no basis for process analysis. Phase space trajectories do tend to be longer, with lower residence time, as energy increases. But this trend does not indicate that an irreversible process has occurred. Debates between Boltzmann and Loschmidt about reversibility and Zermelo and Poincaré about recurrence highlight that deterministic mechanics cannot produce irreversible processes [15]. An isolated deterministic system oscillates unless some artificial mechanism, such as molecular chaos or coarse graining, disrupts the trajectory to achieve equilibration.

This approach of “apparent irreversibility” simply shifts the mystery why systems equilibrate from entropy to state probability without clarification. While it is reasonable to say that differing residence time suggests a dynamical asymmetry, this view still does not explain why some states develop relatively long residence time rather than the system oscillating according to Lagrangian mechanics. Einstein’s statement is intuitive by the common notion of probability, but the concept of state probability is problematic for the following reasons.

In equilibrium, measurement of stable residence time requires that every microstate is tested (ergodicity), that evolution is unitary (quantum mechanics omitting spontaneous events), that probability is single-valued (equal probability postulate), and that the universe is steady during the measurement period. The first three assumptions are plausible, in that they produce satisfactory equilibrium results, but remain generally unproven even in ideal conditions.

These requirements are implausible out of equilibrium because the environment of every particle evolves while approaching equilibrium. Just one spontaneous event somewhere in the system or its environment is sufficient to knock the system state off its ideal unitary course. Then residence time becomes a multi-valued function of phases space, depending upon its history from the starting state. Repeated experiments from every possible initial state would yield inconsistent mean behavior because expected residence time and the settling time to equilibrium varies as the system evolves. Also, as we now have control to create a specific initial state only in very simple microsystems such as qubits, specific non-equilibrium states are not accessible in complex systems or in natural conditions. Therefore, any conclusion about state probability is circular, requiring an assumption about initial distribution. Residence time is not a reliable concept out of equilibrium and there is no known alternative definition for state probability.

These statements relying on the Boltzmann (or similar Gibbs) principle apply only to equilibrium, when the ensemble distribution is stationary. This principle is a static concept due to the extremely long time needed to ascertain state probability, averaging over all dynamics as a result. Comparison of two equilibrium states does indicate what outside exchange must occur in a quasistatic process connecting them, but yields no insight why this happens microscopically. Irreversible processes from a non-equilibrium state to the overwhelmingly probable equilibrium can only be presumed to have occurred by this principle with no insight about their character.

A final point is that Einstein's analysis of opalescence in Ref. [14] does not apply to Onsager's argument. Einstein expressed density fluctuations as a set of independent oscillatory modes and related mode amplitude correlations to the temperature by the equipartition theorem. He then assumes that the local regions remain in quasi-equilibrium, with uniform temperature and well-defined pressure and isothermal compressibility, in order to quantify work done during a fluctuation. Einstein thereby limits his discussion to an initial state with local regions mutually out of equilibrium evolving by a quasi-static process toward system-wide equilibrium. The system correlation length is large enough near critical points where opalescence is observed to justify description of these regions as locally thermalized subsystems with imbalanced variation.

III. FLUCTUATION-DISSIPATION THEOREM

Harry Nyquist made a similar argument in 1928 describing Johnson noise, three years prior to Onsager. He states that "it follows directly from the second law of thermodynamics that the power flowing in one direction is exactly equal to that flowing in the other direction" [2]. Specifically, in equilibrium, current flowing from one macroscopic element, such as an electrical resistor, must be matched by equal power flow to that resistor. A form of the FDT follows by assuming an empirical dissipative transport law relates applied force proportionally to system response by a macroscopic impedance.

Nyquist envisions electromotive charge fluctuations in one resistor inducing current that dissipates in another, and *vice versa*. Macroscopic current flow through a resistor generates heat and consequently increases entropy regardless of direction. Therefore, entropy of an isolated circuit would increase with each fluctuation. Entropy then would not be a state function when other properties are steady.

Furthermore, this description does not match the measurement of Johnson noise. First, Nyquist neglects heat flow to the ambient bath containing the resistor as well as continual exchange with the power supplies driving the circuit, which disrupts the flow symmetry he pictures. Second, the entire circuit is not in equilibrium when the resistor and amplifier and thermocouple ammeter are maintained at different temperatures or drawing power from the environment. Fluctuations then should not be in detailed balance.

Callen and Welton show that the average rate of work done by a perturbing electric potential applied to a circuit by a power supply has a similar form as the unperturbed expectation value of the variance of rate of change in electric dipole moment [5]. They argue that this work is absorbed in the system and so must be dissipation. It is quadratic in the perturbation and therefore conforms to classical linear response theory in which "it is possible to define an impedance, the ratio of force to response..." [5, p. 35]. They apply this presumed impedance formula to the unperturbed dipole variance to produce a general form of Nyquist's formula. (Onsager also assumes a linear response: "In all instances this average expectancy for the rate of transport of heat is related to the momentary distribution by the ordinary macroscopic laws for the conduction of heat" [3, p. 415]. This claim leads to his version of the FDT.)

They state that the "We may expect, even in [an] isolated condition, that the system will exhibit a spontaneously fluctuating [dipole moment], which may be associated with a spontaneously fluctuating force ... [that] does in fact exist" [5, p. 36]. Yet they justify these statements only by assuming the dipole moment obeys an empirical impedance relation to infer a variance in potential. What generates this force? The power supply generating the applied potential likely introduces noise even when the mean bias is zero. But such noise has the characteristics of the external supply, not the system under investigation.

Callen and Welton actually show that both work and fluctuation are regulated by quantum mode transition rates. One might say that system mode transitions generate the spontaneous force the authors seek. These quantities can be compared if they share the same mode spectrum. An isolated circuit obeys different boundary conditions and mode spectrum than when attached to an amplifier, so their comparison requires some experimental qualification.

Equating work and variance indicates the rate of fluctuation production in a steady state if all of the present fluctuation energy is dissipated by macroscopic transport laws, assuming that its particles obey the Maxwellian distribution of the perturbed system Hamiltonian. Kubo stated later that "The linear response theory has given a general proof of the FDT which states that the linear response of a given system to an external perturbation is expressed in terms of fluctuation properties of the system in thermal equilibrium" [7, p. 255]. This statement is too simple for the following reasons.

A system is linear so long as modes of motion are not coupled. Consider three analytical regimes: elemental, thermodynamic and classical. Physics experiments investigating elemental forces are designed for long correlation time such that quantum collapse of the system wave function occurs primarily through interaction with a measuring device. In this regime, system wave function evolution is unitary during the experiment and the independent modes are system Hamiltonian eigenfunctions. The system behaves linearly and all interaction is conservative in this limit of suppressed random events.

In the classical regime, the correlation time is negligibly short and the aggregate effect of microscopic activity on macroscopic bodies may be represented as known forces in classical Lagrangian analysis of those bodies only, while neglecting other forms of diffusion. If these inferred forces are proportional to position and velocity, this classical analysis is again linear, with independent modes of macroscopic body motion matching the classical Hamiltonian eigenfunctions.

Classical analysis may be practical for some purposes but is not a complete description. Thermodynamic analysis treats all particles in the same manner, whether part of a macroscopic body or diffuse medium. Only then can all flows be understood consistently [11]. Linear response theory does not apply to internally generated fluctuation because, by the reasoning given in the next section, the quantum modes available to a particle depend on the local particle configuration, which evolves in a non-Markovian process with strong feedback and non-unitary jumps.

An unanswered question in the FDT derivation is where does the dissipated energy come from and where does it go? To which system is Kubo referring above? The assumption of a linear impedance relation implies that system modes evolve independently with no power exchange between them. Therefore, either the observed noise is driven from outside (not internal fluctuation), or it should not persist (dissipate to zero) in a linear system.

Classical analysis creates an asymmetric model of energy flows between a relatively large body and ambient medium. The mechanism that accelerates the body (fluctuation in the medium) is characterized differently from how energy transfers to the medium (dissipation). This distinction is necessary to allow classical modes to absorb energy in a steady state. The medium is not treated as part of the system, instead as a generic reservoir that conserves energy by both doing work on the body and sinking dissipation from it.

The system that Kubo considers is just one body interacting with a surrounding medium. This combination cannot establish a steady state as the body wanders through the medium. (A large population of similar bodies, mutually coupled through the medium, is needed to form a second material phase before an equilibrium condition of coexisting stable distributions can be resolved within a practical time period.) The medium itself may approximate equilibrium only if its internal thermalization rate is fast and all exchange with the body is relatively very slow. A mesoscopic or larger body can satisfy these conditions. So Kubo refers to separate systems, one macroscopic and the other thermodynamic, that interact out of equilibrium by empirical inference.

This approach is suitable for assessing Brownian motion but provides no explanation why or how dissipation occurs. The measurable trajectory of an individual large body through a medium (or a resistor moving relatively through a medium of electron current in Johnson's experiment) indicates both acceleration and assumed friction forces on the body. Zero net work in a steady state then relates fluctuation amplitude and impedance without reference to detailed balance or the Second Law.

Note that this dynamical asymmetry does not exist for a single phase system. For examples, opalescence is visible scattering of density fluctuations and does not exhibit dissipation, and an isolated resistive material fluctuates but does not by itself generate dissipation in equilibrium. The sequence producing a chance alignment, e.g. in the form of a dipole moment, continues randomly and destroys the alignment just as quickly without altering the mean system properties or producing mean flow of energy. Fluctuation peaks diffuse far more quickly and easily back into the local valleys resulting from their formation rather than motivate high-inertia flow through the entire system.

IV. RIGOROUS DESCRIPTION OF DIFFUSION AND FLUCTUATION

These explanations resort to the Second Law and empirical relations for justification because the concept of flow is not well defined. It is necessary to specify what is actually measured and under what conditions before drawing conclusions. In the thermodynamic regime, encompassing all but unnaturally isolated microsystems, mean measured properties are distinct from fluctuating variation about this mean [16]. Particle trajectories are non-unitary, uncorrelated, and non-laminar due to random events in the ambient medium, and so mix to produce diffusion. (Evolution is unitary only between state reduction events.) Processes beginning out of equilibrium are always irreversible because diffusion incessantly drives a system toward a steady state such that the process appears spontaneous. Any attempt to recover a nonequilibrium state inevitably veers away to a new equilibrium state governed by the current parameters.

The mean and variance of a system property both depend on the measurement time. Measurements much shorter or longer than the system correlation time and length are erratic or obscure underlying trends, respectively. Choosing an appropriate measurement interval separates random variation from systematic behavior. In order to discuss thermodynamics coherently, all terms implicitly reference the measurement period.

We associate macroscopic characteristics of the system with mean measured values. Incessant transition by individual particles among quantum modes produces diffusive flux from any subsystem to another. This microscopic activity also causes flux to fluctuate. Flux occurs simultaneously in reverse direction because mode coupling is bidirectional. Diffusive flux always drives any system toward a steady state of zero net mean flux, in which net aggregate particle motion is balanced in all respects on average over time between all arbitrarily defined subsystems. Such states persist so long as the environment remains steady such that the outside exchange rate is small relative to internal thermalization rate, consistent with our observation of equilibrium. Imbalance causes measurable flow that appears spontaneous only because measurement does not register these countervailing currents separately. Net flows obey transport equations that match known empirical natural laws under appropriate approximation and exhibit reciprocal relations if the partition function is analytic in the conjugate parameters. Therefore, mode transitions account for fluctuation, diffusion, dissipation, the observed "natural laws," and all thermodynamic relations completely without postulate or assumption beyond the standard model of physics [11].

System macroscopic energy and heat content become distinct quantities for measurement time longer than the system correlation time. All particles would follow the same trajectory in a state with only macroscopic energy and no heat. Alternatively, zero macroscopic energy implies no synchronization. Systems in which many particles share macroscopic characteristics tend to transition into less synchronized states, and so appear to dissipate. In other words, friction is the mean effect of net diffusive flux converting macroscopic energy into heat. The reverse is highly unlikely because there are many more less-synchronized states to transition to at any moment. The relation between parameter imbalance and flow is linear, with diffusion coefficient representing the impedance, when imbalance is small.

Absent such conversion, a system evolves as separate conservative classical and thermodynamic subsystems: Any change in system parameters causes microscopic energy (heat and formation energy) to diffuse toward the new steady state equilibrium while macroscopic parameters follow frictionless Lagrangian equations of motion. If the system parameters are then set to a prior configuration, diffusion drives the thermodynamic subsystem back to the corresponding stable state. Any thermodynamic process is "macroscopically reversible" if dissipation is negligible.

Thus we have arrived at a conclusion opposite to the commonly held view that microscopic reversibility somehow appears to be macroscopically irreversible. Instead, particle motion is inherently nonunitary in natural conditions yet systems may be macroscopically reversible if there is no exchange between macroscopic and microscopic energy reservoirs.

Fluctuations are the deviation of properties from their mean macroscopic value. Repeated measurement indicates fluctuation variance. The random nature of mode transitions causes temporary imbalance of diffusive flux producing local accumulation or reduction of energy, momentum and particles. This action necessarily draws from or pushes to surrounding (spatial or energetic) regions of the mode spectrum. Both growth and reduction of fluctuations shift heat content, but with zero net transfer over time. No conversion of macroscopic energy occurs due to fluctuations and so there is no dissipation. On one hand, this comment preserves entropy as a mean state function. Yet on the other hand, fluctuations are not related to dissipation by definition. Therefore, fluctuations play no analytical role other than initiating phase transitions and Brownian motion of mesoscopic bodies.

Fluctuation growth and decay rates are equal in this approach because mode transitions are bidirectional. By assuming sudden growth not just in local domains but also across the system, Onsager in effect selects time intervals from the point when mean fluctuation amplitude peaks until it crosses the system mean value and disregards all other activity. This selection is tantamount to Boltzmann's implicit assumption that a system is initially at peak coherence (maximum of the H-function) in his rebuttal to the reversibility and recurrence objections, because only in that case would entropy never decrease in a deterministic theory [15, Section 4.5]. It assures simply by definition that mean fluctuation amplitude approaches zero in the selected intervals. Sudden growth is an unrealistic restriction designed to achieve the desired conclusion. Onsager's argument would fail if a more complete time interval were selected because then mean fluctuation amplitude would not always regress.

In Statistical Mechanics, equilibrium is maximum disorder (consistent with system constraints), according to the Second Law, and any deviation from equilibrium has a higher degree of order. Then it is reasonable to associate this order with a macroscopic property that decays by dissipation. Onsager models this process by linear response theory, as if this macroscopic motion is the dominant action and all other activity produces no effect but a weak constant resistance. The dynamic asymmetry of this regime breaks detailed balance underlying Onsager's argument. Assumption of linear response and selection of the decay interval are both necessary in his derivation of reciprocal relations but neither choice is consistent with equilibrium balance.

Stated succinctly, diffusion is the mean result of random quantum events, fluctuation is the sum of random events net of diffusion, and thermal flow is the result of differing diffusion rates among subsystems expressed through gradients in thermal properties and parameters. Fluctuation represents a shift in energy, momentum and particles that is randomly oriented relative to these gradients and that averages to zero over time.

Consider the case where flow and fluctuation may be confused: those rare fluctuations involving a very large number of particles spanning a large system in equilibrium. Imagine dividing this system into two subsystems that appear to exchange energy between them as the fluctuation grows and weakens. If this were macroscopic thermal flow, exchange would diminish until both were uniform and steady. Yet fluctuation dynamics must be distinctly different, instead driving each subsystem past this equilibrium state in a repetitive manner. This marginal activity reflects quantum uncertainty, not a response to an applied force. The rate of apparent exchange is oscillatory and insensitive to gradients in thermal properties that drive thermal flow. This explains why fluctuations do not dissipate. Conversely, bulk flow then cannot be ascribed characteristics of fluctuation dynamics in or out of equilibrium and reciprocal relations cannot be justified by detailed balance of fluctuation.

The suggestion that fluctuation can drive flow confuses their complementary nature in this analysis. Of all the particle, momentum and energy exchange implied by the variation in two successive measurements, only the mean trends of many successive measurements are predictable in the form of transport equations. Flow produces a sustained change in the system state, fluctuation does not. Fluctuation only adds uncertainty in a measurement.

V. CONCLUSION

The FDT highlights a coincidence in certain conditions when the ability to discern fluctuation overlaps the validity of a macroscopic approximation of flow. It is limited to experiments on individual mesoscopic objects in contact with a fluid. Acceleration of the object and dissipation are separate effects occurring sequentially in these cases, involving equal power in a steady but non-equilibrium state. There is no basis for assuming such an overlap exists for all processes or assuming that fluctuations decay on average like macroscopic laws of nature. Furthermore, dissipation is invoked only by association with the response to an applied force. While it is reasonable from experience to guess that mesoscopic motion would be damped by friction, this connection is not proven in any derivation the FDT. In each case a "natural law" is assumed to achieve a known result.

Critically, there is no dissipation in equilibrium yet all properties continue to fluctuate. By assuming that macroscopic transport laws apply, the initial state Onsager evaluates is inherently out of equilibrium and therefore not in detailed balance. There is no justification why mean flows (indicating non-equilibrium) should obey the detailed balance of equilibrium fluctuations, and consequently why mean flows should obey reciprocal relations.

Diffusion and fluctuation are both symptoms of random quantum events, but one does not imply the other. This conclusion becomes clear when rigorous definition of measurement distinguishes trajectory mean and deviation. Then diffusion and dissipation are derived directly from mean parameters without reference to fluctuation. No additional postulates or assumption beyond the standard model of physics are necessary to derive all transport laws. Near-equilibrium reciprocal flow relations exist whenever the partition function is analytic in two or more system parameters.

In this perspective, flow and fluctuation have different sources. Net flow is driven by mean property gradients, as understood in empirical natural laws, and diminishes to zero when the mean state is uniform in equilibrium. Random quantum events produce temporary bidirectional deviation from mean behavior. Fluctuations then represent net displacement of many independent particle shifts, not macroscopic transfer caused by a common driver. Fluctuation and diffusion are complementary effects with distinct dynamics, which explains why fluctuations neither dissipate nor enter thermodynamic process equations.

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